Hans Wackernagel

Multivariate Geostatistics

An Introduction with Applications

Third Edition

Springer

Hans Wackernagel

Multivariate Geostatistics

An Introduction with Applications

Third, completely revised edition

with 117 Figures and 7 Tables

UNIVERSITÀ D. STUDI DI TRENTO ★ BIBLIOTECA

Springer

Contents

1 **Introduction** 1

A From Statistics to Geostatistics 7

2 **Mean, Variance, Covariance** 9
The mean: center of mass . . . 9
Distribution function . . . 11
Expectation . . . 12
Variance . . . 13
Covariance . . . 14

3 **Linear Regression and Simple Kriging** 15
Experimental covariance . . . 15
Linear regression . . . 16
Variance-covariance matrix . . . 20
Multiple linear regression . . . 21
Simple kriging . . . 24

4 **Kriging the Mean** 27
Arithmetic mean and its estimation variance . . . 27
Estimating the mean with spatial correlation . . . 28
No systematic bias . . . 29
Variance of the estimation error . . . 30
Minimal estimation variance . . . 30
Kriging equations . . . 31
Case of no correlation . . . 32

B Geostatistics 35

5 **Regionalized Variable and Random Function** 39
Multivariate time/space data . . . 39
Regionalized variable . . . 40
Random variable and regionalized value . . . 41

Random function . . . 41
Probability distributions . . . 42
Strict stationarity . . . 43
Stationarity of first two moments . . . 44

6 Variogram Cloud **45**
Dissimilarity versus separation . . . 45
Experimental variogram . . . 47
Replacing the experimental by a theoretical variogram . . . 48

7 Variogram and Covariance Function **50**
Regional variogram . . . 50
Theoretical variogram . . . 50
Covariance function . . . 52
Positive definite function . . . 53
Conditionally negative definite function . . . 53
Fitting the variogram with a covariance function . . . 55

8 Examples of Covariance Functions **57**
Nugget-effect model . . . 57
Exponential covariance function . . . 57
Spherical model . . . 58
Derivation of the spherical covariance . . . 58

9 Anisotropy **62**
Geometric Anisotropy . . . 62
Rotating and dilating an ellipsoid . . . 62
Exploring 3D space for anisotropy . . . 64
Zonal anisotropy . . . 65
Nonlinear deformations of space . . . 65

10 Extension and Dispersion Variance **66**
Support . . . 66
Extension variance . . . 67
Dispersion variance . . . 68
Krige's relation . . . 69
Change of support effect . . . 70
Change of support: affine model . . . 71
Application: acoustic data . . . 73
Comparison of sampling designs . . . 76

11 Ordinary Kriging **79**
Ordinary kriging problem . . . 79
Simple kriging of increments . . . 81
Block kriging . . . 82

Simple kriging with an estimated mean . . . 84
Kriging the residual . . . 85
Cross validation . . . 87
Kriging with known measurement error variance . . . 88

12 Kriging Weights **89**
Geometry . . . 89
Geometric anisotropy . . . 91
Relative position of samples . . . 91
Screen effect . . . 92
Factorizable covariance functions . . . 93
Negative kriging weights . . . 94

13 Mapping with Kriging **96**
Kriging for spatial interpolation . . . 96
Neighborhood . . . 97

14 Linear Model of Regionalization **101**
Spatial anomalies . . . 101
Nested variogram model . . . 102
Decomposition of the random function . . . 103
Second-order stationary regionalization . . . 104
Intrinsic regionalization . . . 105
Intrinsic regionalization with mostly stationary components . . . 105
Locally stationary regionalization . . . 106

15 Kriging Spatial Components **107**
Kriging of the intrinsic component . . . 107
Kriging of a second-order stationary component . . . 108
Filtering . . . 110
Application: kriging spatial components of arsenic data . . . 111

16 The Smoothness of Kriging **113**
Kriging with irregularly spaced data . . . 113
Sensitivity to choice of variogram model . . . 115
Application: kriging topographic data . . . 117

C Multivariate Analysis **121**

17 Principal Component Analysis **123**
Uses of PCA . . . 123
Transformation into factors . . . 123
Maximization of the variance of a factor . . . 125
Interpretation of the factor variances . . . 126
Correlation of the variables with the factors . . . 127

18 Canonical Analysis 137
Factors in two groups of variables . . . 137
Intermezzo: singular value decomposition . . . 138
Maximization of the correlation . . . 138

19 Correspondence Analysis 140
Disjunctive table . . . 140
Contingency table . . . 140
Canonical analysis of disjunctive tables . . . 141
Coding of a quantitative variable . . . 141
Contingencies between two quantitative variables . . . 141
Continuous correspondence analysis . . . 142

D Multivariate Geostatistics 143

20 Direct and Cross Covariances 145
Cross covariance function . . . 145
Delay effect . . . 145
Cross variogram . . . 146
Pseudo cross variogram . . . 149
Difficult characterization of the cross covariance function . . . 150

21 Covariance Function Matrices 151
Covariance function matrix . . . 151
Cramer's theorem . . . 151
Spectral densities . . . 152
Phase shift . . . 153

22 Intrinsic Multivariate Correlation 154
Intrinsic correlation model . . . 154
Linear model . . . 155
Codispersion coefficients . . . 156

23 Heterotopic Cokriging 158
Isotopy and heterotopy . . . 158
Ordinary cokriging . . . 159
Simple cokriging . . . 161

24 Collocated Cokriging 165
Cokriging neighborhood . . . 165
Collocated simple cokriging . . . 166
Collocated ordinary cokriging . . . 167
Simplification with a particular covariance model . . . 168

25 Isotopic Cokriging 170
Cokriging with isotopic data . . . 170
Autokrigeability . . . 171
Bivariate ordinary cokriging . . . 173

26 Multivariate Nested Variogram 175
Linear model of coregionalization . . . 175
Bivariate fit of the experimental variograms . . . 177
Multivariate fit . . . 178
The need for an analysis of the coregionalization . . . 181

27 Case Study: Ebro Estuary 183
Kriging conductivity . . . 183
Cokriging of chlorophyll . . . 185
Conditional simulation of chlorophyll . . . 187

28 Coregionalization Analysis 194
Regionalized principal component analysis . . . 194
Generalizing the analysis . . . 195
Regionalized canonical and redundancy analysis . . . 196
Cokriging regionalized factors . . . 196
Regionalized multivariate analysis . . . 197

29 Kriging a Complex Variable 200
Coding directional data as a complex variable . . . 200
Complex covariance function . . . 200
Complex kriging . . . 201
Cokriging of the real and imaginary parts . . . 202
Complex kriging and cokriging versus a separate kriging . . . 203
Complex covariance function modeling . . . 205

30 Bilinear Coregionalization Model 207
Complex linear model of coregionalization . . . 207
Bilinear model of coregionalization . . . 208

E Selective Geostatistics 211

31 Thresholds and Selectivity Curves 213
Threshold and proportion . . . 213
Tonnage, recovered quantity, investment and profit . . . 213
Selectivity . . . 216
Recovered quantity as a function of tonnage . . . 217
Time series in environmental monitoring . . . 219

32 Lognormal Estimation 221
Information effect and quality of estimators . . . 221
Logarithmic Gaussian model . . . 223
Moments of the lognormal variable . . . 224
Lognormal simple kriging . . . 226
Proportional effect . . . 228
Permanence of lognormality . . . 228
Stable variogram model . . . 233
Lognormal point and block ordinary kriging . . . 233

33 Gaussian Anamorphosis with Hermite Polynomials 238
Gaussian anamorphosis . . . 238
Hermite polynomials . . . 239
Expanding a function into Hermite polynomials . . . 240
Probabilistic interpretation . . . 240
Moments of a function of a Gaussian variable . . . 241
Conditional expectation of a function of a Gaussian variable . . . 243
Empirical Gaussian anamorphosis . . . 244
Smoothing the empirical anamorphosis . . . 245
Bijectivity of Gaussian anamorphosis . . . 247

34 Isofactorial Models 250
Isofactorial bivariate distribution . . . 250
Isofactorial decomposition . . . 251
Isofactorial models . . . 252
Choice of marginal and of isofactorial bivariate distribution . . . 253
Hermitian and Laguerre isofactorial distributions . . . 254
Intermediate types between diffusion and mosaic models . . . 258

35 Isofactorial Change of Support 262
The point-block-panel problem . . . 262
Cartier's relation and point-block correlation . . . 262
Discrete Gaussian point-block model . . . 266
General structure of isofactorial change-of-support . . . 267

36 Kriging with Discrete Point-Bloc Models 273
Non-linear function of a block variable . . . 273
Conditional expectation and disjunctive kriging of a bloc . . . 274
Disjunctive kriging of a panel . . . 275
Uniform conditioning . . . 277

F Non-Stationary Geostatistics 281

37 External Drift 283
Depth measured with drillholes and seismic . . . 283
Estimating with a shape function . . . 284
Estimating external drift coefficients . . . 285
Cross validation with external drift . . . 290
Regularity of the external drift function . . . 294
Cokriging with multiple external drift . . . 296
Ebro estuary: numerical model output as external drift . . . 297
Comparing results of conditional simulations and kriging . . . 297

38 Universal Kriging 300
Universal kriging system . . . 300
Estimation of the drift . . . 302
Underlying variogram and estimated residuals . . . 303
From universal to intrinsic kriging . . . 306

39 Translation Invariant Drift 308
Exponential-polynomial basis functions . . . 308
Intrinsic random functions of order k . . . 309
Generalized covariance function . . . 310
Intrinsic kriging . . . 311
Trigonometric temporal drift . . . 312
Filtering trigonometric temporal drift . . . 312
Dual kriging . . . 313
Splines . . . 314

APPENDIX 317

Matrix Algebra 319

Linear Regression Theory 329

Covariance and Variogram Models 334

Additional Exercices 337

Solutions to Exercises 339

References 353

Bibliography 358

Index 381